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POINTED TWO-PARAMETER POWER-LAW NOSE SHAPES OF MINIMUM WAVE DRAG†

S. A. TAKOVITSKII

Zhukovskii

e-mail: c.a.t@tsagi.ru

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A direct method of constructing pointed contours which are close to optimum with respect to wave drag is developed for axisymmetric nose shapes using Euler's equations. A two-parameter power function is a good approximation of the contours constructed using this method. Calculations, carried out using the proposed approximation, demonstrate the reduction in the wave drag of the bodies constructed compared with existing optimum, blunt, one-parameter, power-law nose shapes. © 2003 Elsevier Ltd. All rights reserved.

One of the classical problems of supersonic aerodynamics is to determine axisymmetric nose shapes which ensure the minimum wave drag for specified constraints on the overall size. The first optimum nose shapes, found by Newton, are characterized by the existence of a front face. Investigations within the framework of Newton's and Busemann's drag laws have revealed that the front face is a section of a boundary extremum [1]. In the approximation of the Newton–Busemann formula, a second feature of the optimum contour arises, that is, its closing section in which the gas pressure is equal to zero [2]. A solution of the same problem within the framework of linear theory was obtained by Karman [3]. The use of analytic relations, close to those obtained using the method of tangent cones, for the pressure distribution on the body surface has enabled nose shapes to be designed for which a significant reduction in the drag compared with certain pointed shapes has been confirmed experimentally [4].

Aerodynamic shapes, obtained using a simplified formulation of a variational problem, have sections for which the pressure distribution is calculated with a large error. Nevertheless, the efficiency of the approximate laws for the profiling nose shapes has been confirmed by investigations using exact methods of calculation. In the case of single-parameter, power-law bodies ($r \sim x^m$, where x is measured along the axis of symmetry from the leading point of the body and r is the distance to the axis), calculations using Euler's equations showed that the optimum value of the exponent m lies in the range 0.60 to 0.75 [5]. Numerical optimization investigations make large demands on computer time and are carried out with a constraint on the configurations considered with the aim of reducing the number of independent parameters [6].

When solving the problem using Euler's equations, it has been established [7–12] that the unique, pointed, axisymmetric nose section, past which there is a flow with an attached shock wave, is a needle (a segment of the axis of symmetry) which is of no interest. Axisymmetric nose shapes, differing from needles and around which there is a flow with an attached shock wave of finite strength, are bodies with a channel. Optimum axisymmetric nose shapes without a channel produce a detached shock wave. This enables us to assume that they, as within the framework of the approximate models, have a front face, that is, a boundary extremum which appears due to the constraint imposed on the length of the nose section. Recent investigations show [13] that this is actually so and a large number of examples of the construction of optimum nose shapes with a front face are given.

When the aspect ratio λ (the ratio of the length to the diameter of the base) is increased, the optimum, size of the front face rapidly decreases [13]. Thus, for $\lambda = 2$ and Mach numbers M = 3 ... 10, the radius of the front face is 0.08 ... 0.06 of the radius of the base, becoming even smaller as λ increases. The fact that there is a reduction in the relative dimensions of the front face and its contribution to the drag when the body thickness is reduced suggests that pointed bodies can be close to optimum bodies as regards their shape if the aspect ratios are sufficient. The optimum form of the generatrix for bodies with a sharp vertex is determined below by the direct method of optimization with a large number of geometrical parameters ($n = 350$), and it is shown that it is well approximated by a two-parameter power-law relation. The superiority of two-parameter, pointed power-law bodies over single-parameter, blunt

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bodies over the whole range of free-stream Mach numbers M = 1.5, … , 4 has been established by systematic investigations of power-law bodies using Euler's equations.

1. FORMULATION OF THE PROBLEM AND THE DIRECT METHOD OF OPTIMIZATION

It is required to find the absolute minimum of the function

$$X(r_1, r_2, \dots, r_n) = \min$$

which represents the dependence of the wave drag X on the geometrical parameters r_i ($i = 1, \dots, n$). The problem was solved in two stages which differ in the number of parameters.

In the first stage, investigations were carried out using a large number of parameters, which enabled us to widen the class of configurations being considered to the greatest extent. The radii of the nodal cross-sections were chosen as the geometrical parameters. There were $n = 350$ of them. The nodal cross-sections condensed towards the first cross-section (closest to the vertex of the nose section, coinciding with the origin from which x is measured) in accordance with the relation

$$x_i = x_1 + \frac{s^{i-1} - 1}{s^n - 1}(L - x_1)$$

Here, x_i is the distance from the vertex of the body to the cross-section with serial number i, L is the body length and $s = 1.012$ is the condensation factor. The first cross-section is at a distance from the vertex of the body of 1% of the body length ($x_1 = 0.01L$). The generatrix of the body was represented by a set of sections joining neighbouring nodal cross-sections and the base of the nose section. The following relation between the radius and the longitudinal coordinate was adopted for the section of the generatrix in front of the first cross-section: $x \equiv \varphi(r) = Fr + Gr^2$. The coefficients F and G were determined by the conditions of smoothness of the generatrix with respect to the values of r_1 and r_2, that is, the radii of the first and second nodal cross-sections, admitting of configurations both with a convex as well as with a concave generatrix.

In the second stage, after the features of the geometry of the optimum bodies had been established, the number of parameters was reduced to two. The radius of the generatrix as a function of x was approximated by a power function, which, in the general case, gives a pointed nose shape.

A method first employed when optimizing an isolated wing and a wing which is interacting with other parts of an aircraft [14, 15] was used to solve the problem. It is based on establishing the relation between the gas dynamic functions and the geometrical parameters within the framework of a local analysis. This analysis rested on a linearization which assumes local closeness of the flow to supersonic plane-parallel flow (the "local Ackeret formula"). As a result, the approximation of the objective function (the drag) is determined as a quadratic form. Information concerning the gradient and Hessian matrix of the function ensures rapid convergence to the optimum. Shape variations, directed towards reducing the drag, are found using Newton's method.

When optimizing a body of revolution, the method was modified in the following manner. The local analysis was carried out for a constant radius r_1 of the generatrix in the first nodal cross-section. The extremum value of r_1 was determined by an additional variation of the shape. In this case, the increments in the radii when $i = 2, \dots, n$ are connected by the linear relation

$$\Delta r_i = \Delta r_1 \frac{L - x_i}{L - x_1}$$

The optimization cycle therefore consisted of a descent along two directions in the n-dimensional space.

In the second stage, the problem was solved by the method of cyclic coordinate descent. The minimum number of parameters and their fortunate choice (on the basis of the topographical features of the objective function) determined the efficiency of this approach.

The flow of a perfect gas about a body of revolution was calculated by the marching method [16]. The discontinuity in the gas dynamic variables in the bow shock wave was rigorously isolated. Euler's equations were integrated using an explicit McCormack finite-difference scheme. Flows with an attached bow shock wave, the flow beyond which remains supersonic in the direction of the axis of symmetry, were investigated. The maximum number of mesh points in the computational mesh between the body surface and the shock wave reached 400.

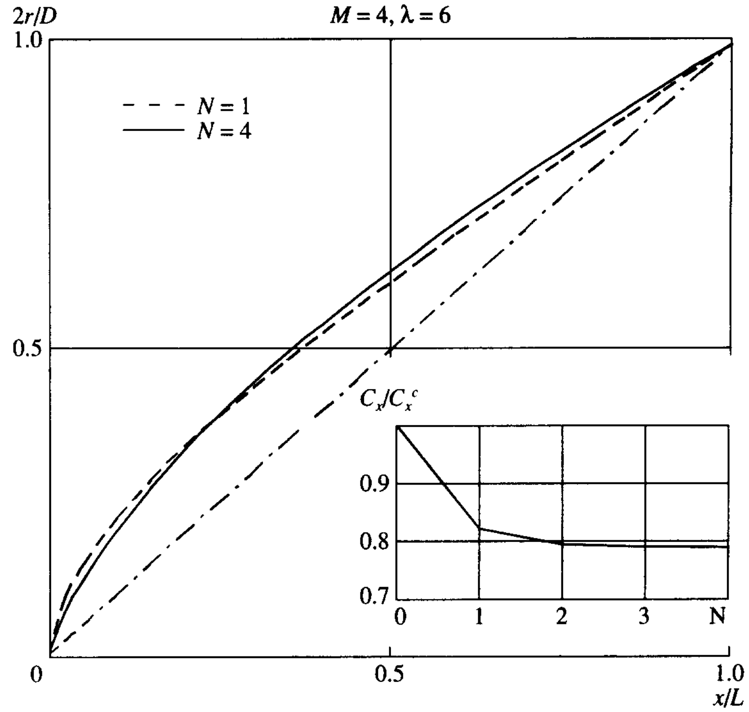

Fig. 1

The choice of the nodal cross-sections which has been described above ensures that the inequalities $x_i - x_{i-1} \ll r_i$ are satisfied for $i \geq 2$, which enable one to use a "plane-parallel linearization" and, as a consequence of this, lead to the correct determination of the direction of decrease in X in the space of the parameters $r_2, \ldots, r_n$. Without such a choice, with a uniform arrangement of all the nodal cross-sections along the axis of symmetry, for example, the direction of the decrease in X, even in the same parameter space $r_2, \ldots, r_n$, would be incorrectly determined and the method of direct optimization would not work. In the case of sufficiently high aspect ratios (depending on M), the choice of x_1, s and $\varphi(r)$ is restricted from above and the resulting "optimum" inclination of the initial portion (or the ratio r_1/x_1) is such that the flow past the tip turns out to be supersonic. As the aspect ratio is reduced (which corresponds to an increase in the size of the front face [13]), subsonic flow occurred despite the measures which had been taken, and the approach which had been developed ceased to work.

2. RESULTS OF THE INVESTIGATION

The investigation was carried out over a range of supersonic velocities corresponding to Mach numbers M = 1, 5, 2, 3 and 4. Optimum bodies with an aspect ratio $\lambda = L/D = 2, 3, 4, 5, 6$ and 10 (D is the diameter of the base) were determined. A body with $\lambda = 2$ when $M \leq 2$ was not considered. The area of the base was taken as the characteristic area when calculating the wave drag coefficient C_x.

In the first stage of the investigations, a method was used which is based on Newton's method and ensures exceptionally rapid convergence to the optimum. In spite of the large number of parameters which are varied ($n = 350$), the optimum shape of the body is determined after just four cycles. The change in the generatrix and the drag of the nose section (divided by the drag C_x^c of cone) during the optimization of a body with $\lambda = 6$ at $M = 4$ are shown in Fig. 1.

The initial generatrix was conical (the dotted and dashed line corresponds to it) but just the first optimization cycle ensured that the shape fell into the immediate neighbourhood of the optimum. The reduction in C_x after the first cycle was 18% and, after the final cycle (when $N = 4$), it was 21%.

For the same $\lambda = 6$ and $M = 4$, a comparison of the generatrix obtained by numerical optimization with the contour of a blunt, single-parameter, optimum nose section of power-law shape [5], revealed the closeness between their shapes when $x > 0.05L$. The wave drag coefficients also have close values of 0.0183 and 0.0185, respectively. However, in the case of lower Mach numbers and higher aspect ratios, a more detailed approximation of the nose shape is required. Using the results of an analysis of optimum

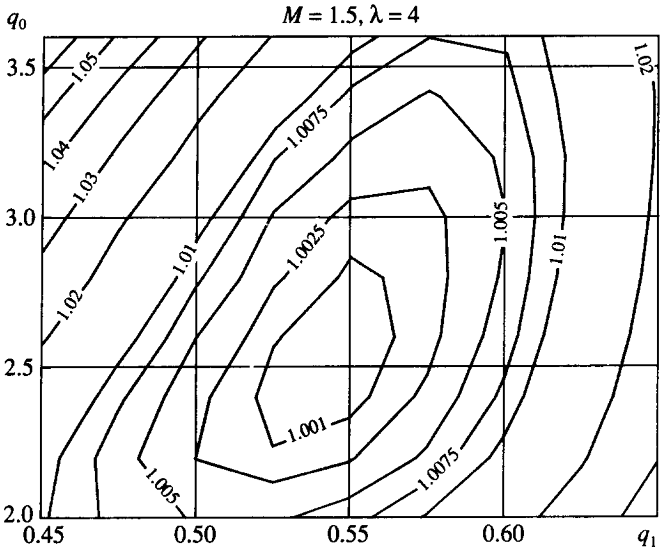

Fig. 2

shapes constructed using direct numerical optimization, it has been shown that a two-parameter power-law relation of the form

$$2r/D = B[1-(1+Ax/L)^m] \tag{2.1}$$

is a good approximation of their generatrices.

With this relation, the maximum error when calculating the radius does not exceed 3%.

The use of an analytical representation of the generatrix of a body enabled us to reduce the number of parameters to the maximum permissible extent and to carry out systematic optimization investigations. The body shape is completely determined by the specification of two geometrical parameters. An analysis of the topography of the objective function demonstrated the appropriateness of the choice of the derivatives of the radius at the tip and the radius of the tail cross-sections as the parameters. For convenience of representation, the derivatives were normalized in terms of the aspect ratio

$$q_0 = 2\lambda dr/dx|_{x=0}, \quad q_1 = 2\lambda dr/dx|_{x=L}$$

Specification of q_0 and q_1 determines the values of the parameters occurring in representation (2.1): $B = -q_0/(mA)$, and the coefficient A and the exponent m are found from the numerical solution of the following equations

$$1 = -[q_0/(mA)][1-(1+A)^m], \quad q_1 = q_0(1+A)^{m-1}$$

The level lines of the wave drag coefficient (divided by the minimum value) of a body with an aspect ratio $\lambda = 4$ at $M = 1.5$ in the plane of the geometrical parameters (q_0, q_1) are shown in Fig. 2. The significantly weaker effect of the parameter q_0 on the wave drag compared with the parameter q_1 is due to the large contribution to the drag from the peripheral segments of axisymmetric bodies.

The results of the systematic investigations are presented in the form of the graphs of the geometrical parameters q_0 and q_1 and the wave drag coefficients $C_x\lambda^2$ on the supersonic similarity parameter for thin bodies β/λ, where $\beta = \sqrt{M^2-1}$ (Fig. 3). Over the range of M and λ considered, the parameter q_0 for all optimum bodies has values which correspond to flow past a body with an attached bow shock wave. At the same time, the angle of inclination of the generatrix at the tip increases as the aspect ratio is reduced. Detachment of the shock wave is inevitable when λ is reduced further.

Comparison with the data in [5] shows that pointed (two-parameter) and blunt (single-parameter) optimum, power-law bodies have similar aerodynamic characteristics for moderate aspect ratios. At the same time, good agreement is found with respect to the parameter q_1. The equality $q_1 = m$ is satisfied in the case of a single-parameter blunt body. As β/λ is reduced, the difference in the values of the

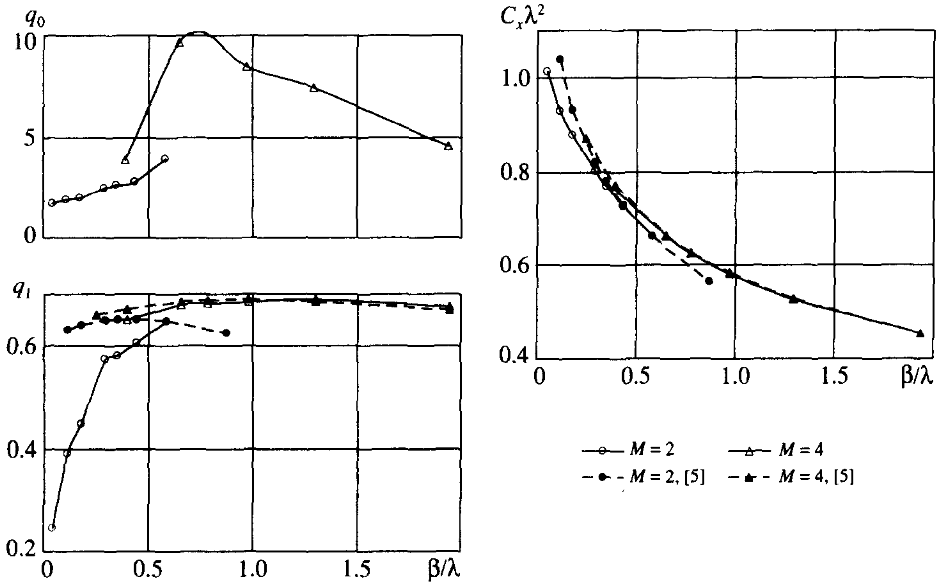

Fig. 3

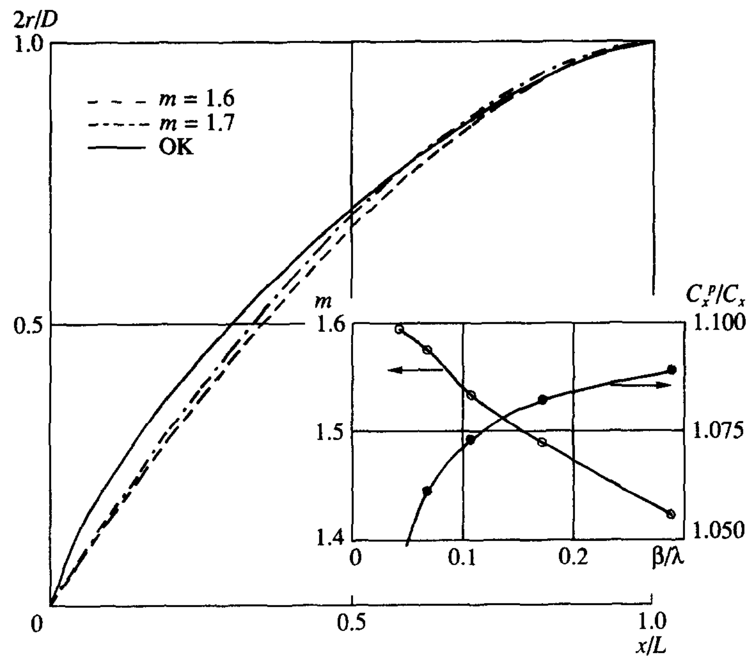

Fig. 4

parameter q_1 and the wave drag increases. For example, when M = 2 and $\lambda = 10$, a pointed, two-parameter, power-law nose section has a wave drag which is 6% less than the blunt nose part from [5].

It has been shown [17] that the drag of bodies of power-law and parabolic form of low aspect ratio can be reduced by introducing spherical blunting of a specified radius. However, for of high aspect ratio this approach leads to the inverse effect. For instance, when M = 1.5 and $\lambda > 5$, the bodies from [17] with spherical blunting have a wave drag which exceeds the theoretical value $C_x\lambda^2 = 1$ in the case of a Karman ogive [3]. In this case, the transition to pointed, two-parameter, power-law bodies enables C_x to be reduced by more than 20%.

Investigations of bodies of very high aspect ratio (with λ values up to 40), when M = 2, were carried out in order to determine the optimum shape of a body of infinitesimal thickness. It was established that there is a monotonic decrease in the geometrical parameters q_0 and q_1 as the similarity parameter

β/λ tends to zero (Fig. 3). If one puts $q_1 = 0$, then the search is confined to the class of "parabolic" contours with an unknown exponent

$$2r/D = 1-(1-x/L)^m$$

The admissibility of this assumption is confirmed by the convergence of the minimum values of the wave drag of bodies with "parabolic" (C_x^p) and power-law contours as β/λ is reduced (Fig. 4). In turn, the extremum value of the exponent m also increases monotonically and, when $\beta/\lambda = 0$, can be confined to the range $1.6 \le m \le 1.7$. The optimum parabolic contour of a body of infinitesimal thickness which has been found is substantially different from a Karman ogive both in the leading and the central portions.

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